photocathode; l, emissivity in middle of spectral sensitivity range of photocathode; $\theta = \alpha/l$, parameter characterizing deviation of emissivity of object from that of a blackbody.

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OPTIMAL GAS DISCHARGE THROUGH CRYOGENIC FLOW INLETS IN REFRIGERATOR COOLING OF SUPERCONDUCTING MAGNETIC SYSTEMS

Yu. L. Buyanov

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A relationship connecting the discharge of the gas-cooling cryogenic flow inlets to the discharge of the gas returning in the reverse flow is obtained, for which the cold-productivity of the refrigerator, which is required, is a minimum.

Cryostating the turns of superconducting magnets (SM) with large working volumes is accomplished, as a rule, by using helium refrigeration apparatus. In the majority of cases, the quantity of liquid helium needed, which is delivered to the SM cryostat, is produced in the refrigerator and the refrigerator cold-productivity is later used to maintain a given level of liquid helium by compensating for the heat influxes into the low-temperature zone.

Part of the cold gas in the refrigerator cooling of SM is ordinarily supplied to cool the flow inlets (this gas flow is heated to the temperature of the environment and is not directed into the reverse flow of the refrigerator).

A diagram of the lower part of the refrigerator including the Joule – Thomson heat exchanger 1, throttle 2, and the SM cryostat 3, is shown in Fig. 1. The flow inlet channel being cooled 4 is also shown.

The liquefiable part M_l of the gas flow M_i passing through the throttle compensates for the decrease in the liquid helium in the cryostat, while the unliquefied helium goes together with the saturated vapor being formed because of the total heat influx ΣQ to the cryostat, into the refrigerator reverse flow M_2 and into cooling the flow inlets M_3 . The total heat influx is comprised of the cryostat background Q_{bgd} (the heat influx to the cryostat from different heat sources), the heat influx over the flow inlets being cooled Q_{cld} (M_3), and the additional heat influx Q_{add} , which can be considered as the refrigerator power reserve; i.e.,

$$\Sigma Q = Q_{\text{bgd}} + Q_{\text{cld}}(M_3) + Q_{\overline{a}dd}$$
(1)

The maximum refrigerator power required in order to cool the SM is defined by the relationship

$$Q_{\rm r} = \Sigma Q + Q_3 \frac{l_{\rm f}}{l_{\rm r}} - Q_3, \qquad (2)$$

where $Q_3 = M_3 r$ is the cold used to cool the flow inlets and the term $Q_3 (l_f/l_r)$ in (2) is the refrigerator cold-productivity ensuring a supply of liquid helium in the amount M_3 .

Using the notation $\gamma = r[(l_f/l_r) - 1]$, we have

$$Q_{r} = \Sigma Q + \gamma M_{3}. \tag{3}$$

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The energy expended in cooling the SM depends on the gas consumption in cooling the inlets. The refrigerator power required increases with the increase in M_3 , but, on the other hand, a diminution in M_3 will result in magnification of the heat influx over the flow inlets $Q_{cld}(M_3)$, which also causes a rise in the cold-productivity of the refrigerator apparatus to be needed. Hence, there exists an optimal value of the gas consumption at which the refrigerator cold-productivity required will be minimal.

Different aspects of the operation of flow inlets in the refrigerator cooling of superconducting magnetic systems have been examined by a number of authors [1-3]. This paper is devoted to the question of determining a relationship between the gas being returned in the refrigerator reverse flow and the gas being delivered to cool the inlets, which will assure minimal required refrigerator power.

The differential equation

$$\frac{d^2T}{dx^2} - \frac{M_3 c_p}{\lambda(T) S} \frac{d\theta}{dx} + \frac{I^2 \rho(T)}{\lambda(T) S^2} = 0$$
(4)

can be obtained in considering the energy balance of an elementary section dx of the flow inlet.

The solution of (4) can be obtained in analytic form only upon making special assumptions relative to the form of the functions $\lambda(T)$, $\rho(T)$, $\theta(x)$.

Let us represent the change in temperature of the cooling gas in the form of the function

$$\theta = \theta_{cld} (1 + K_{\theta} x), \tag{5}$$

where θ_{cld} is the temperature of the saturated helium vapor at the lower end of the flow inlet and $K_{\theta}^* = K_{\theta}/K_{cld}$ is the axial temperature coefficient.

Then for constant values of λ and ρ , Eq. (4) has the form

$$\frac{d^2T}{dx^2} - \frac{M_3 c_p K_0}{\lambda S} + j^2 \frac{\rho}{\lambda} = 0.$$
(6)

Integrating (6) under the boundary conditions $T|_{x=0} = T_{cld}$ and $T|_{x=l} = T_{hot}$, we obtain

$$T = T_{\rm cld} + \frac{D}{2} x^2 + \frac{x}{l} \left(\Delta T - D \frac{l^2}{2} \right),$$
 (7)

where $D = M_3 c_p K_{\theta} / \lambda S - j^2 (\rho / \lambda)$, and $\Delta T = T_{hot} - T_{cld}$.

Then, the heat supply over the flow inlets in the low-temperature zone is

$$Q_{\text{cld}}(M_3) = \lambda S \left(\frac{dT}{dx}\right)_{x=0} = \frac{\lambda S}{l} \left[\Delta T - \left(\frac{M_3 c_p K_{\theta}}{\lambda S} - j^2 \frac{\rho}{\lambda}\right) \frac{l^2}{2}\right].$$
(8)

The result obtained is also applicable to the case of Dewar cooling of SM, when the heat elimination from the flow inlets is accomplished by the cold gas they generate from the liquid helium.

Setting $Q_{cld} = M_3 r$ into (8), we obtain a simple algebraic dependence governing the heat influx over the flow inlets:



Fig. 2. Characteristic temperature change along the length of a flow inlet operating in the optimal mode T, K;l, m.

Fig. 3. Dependence of the ratio M_3/M_2 on the temperature of preliminary cooling (the parameter is the forward flow pressure). 1) 1.5; 2) 2; 3) 2.5 MPa. T_1 , °K.

$$Q_{\text{cld}} = \left(1 + \frac{c_p K_{\theta} l}{2r}\right)^{-1} \left[\frac{\lambda S \Delta T}{l} + j^2 \frac{\rho l S}{2}\right].$$
(9)

It is seen from (1) and (3) that when $Q_{cld}(M_3) = 0$, the reserve in refrigerator cold-productivity is greatest, or the refrigerator power required to cool the SM is minimal for $Q_{add} = \text{const.}$ It should be noted that in real cases, the total heat influx $Q_{cld}(M_3) + Q_{bgd}$ approaches Q_{bgd} asymptotically with the increase in M_3 and $Q_{cld} \neq 0$ for any values of M_3 . However, the nature of the dependence $Q_{cld}(M_3)$, obtained in testing flow inlets with refrigerator cooling is steep [2], and $Q_{cld}(M_3) \ll Q_{bgd}$ for some value of M_3 ; hence, when the linear function $Q_{cld}(M_3)$ equals zero, the quantity M_3^* obtained from (8) can be used to estimate the greatest reserve in the refrigerator cold-productivity. The growth of Q_{add} with the increase in M_3 is possible under the condition $\gamma > (c_p K_{\theta} l)/2$, where $(c_p K_{\theta} l)/2$ is the absolute value of the angular coefficient of the function $Q_{cld}(M_3)$. Later, when $M_3 > M_3^*$, Q_{add} diminishes to zero for $M_3 = (Q_r - Q_{bgd})/\gamma$.

Then

$$M_3^* = (c_p K_\theta)^{-1} \left(\frac{2\lambda S \Delta T}{l^2} + \frac{l^2 \rho}{S} \right)$$
(10)

or we obtain the least value of M_3^* , for which the required refrigerator cold-productivity will be a minimum,

$$(M_3^*)_{\min} = 2I \left(2LT\Delta T \right)^{1/2} / c_p K_{\theta} l , \qquad (11)$$

from the condition of the minimum of the function $M_3^*(S)$ and, moreover, the use of the Wiedemann – Frantz law ($\lambda \rho = LT$).

The refrigerator cold-productivity can be determined by the following dependence:

$$Q_{r} = M_{i} \left(\Delta i_{T} - c_{p} \Delta t - q_{en} \right), \tag{12}$$

where $\Delta i_T = i_b - i_a$ is the isothermal effect of the throttling on the temperature level T_a , and Δt is the underrecuperation at the warm end of the Joule – Thomson heat exchanger (the letter subscripts of the enthalpy i correspond to the points in Fig. 1).

Substituting (11) and (12) into (3), we obtain the ratio M_3/M_1 ensuring the minimal required refrigerator power for SM cooling for $Q_{cld}(M_3) = 0$:

$$\frac{M_{3}}{M_{1}} = \frac{\Delta i_{T} - (c_{p}\Delta t + q_{en})}{\gamma + [c_{p}K_{\theta}l(Q_{add} + Q_{bgd})]/[2l(2LT\Delta T)^{1/2}]}.$$
(13)

Since $M_2 = M_1 - M_3$, we obtain

$$M_3/M_2 = A/(1 - A),$$
 (14)

where A is the expression in the right-hand side of (13).

The quantity γ in (13) can be obtained from the equation $\gamma = r[(l_f/l_r) - 1]$, which can be written in the form

$$\gamma = (T_{\rm en}\,\Delta s - \Delta i)\,\varepsilon_{\rm C} - r. \tag{15}$$

for an ideal refrigerator operating in a Carnot cycle.

Here Δs and Δi are the differences between the values of the entropy and enthalpy of the working gas at the temperature of the cooling medium and the liquid helium corresponding to the pressure of the refrigerator reverse flow.

The quantity γ is evaluated in [1] and it has been obtained that $\gamma \approx 80 \text{ J/g}$ for a 4.4°K cryostat temperature level.

The coefficient K_{θ}^{*} can be determined in terms of the mean temperature of the cooling gas:

$$\overline{\theta} = (1/l) \int_{0}^{l} \left[\theta_{\text{cld}} (1 + K_{\theta}^{*} x) \right] dx.$$
(16)

We obtain after integrating (16)

$$K_{\theta}^{*} = (2/l) \left(\frac{\overline{\theta}}{\theta_{cld}} - 1 \right).$$

The temperature profile obtained in an experimental investigation of 12 different flow inlets operating in the optimal mode (i.e., in the mode corresponding to a minimal value of the quantity $q_{cld} = Q_{cld}/I$) is shown in Fig. 2. The structure peculiarities, as well as the electrical properties of the material of the current-carrying element of the inlet, are presented in [4]. The temperature of the cooling gas, corresponding to the upper section of the flow inlet, is also superposed in Fig. 2.

Referring the lower curve bounding the temperature profile of the flow inlet to $\theta = f(x)$, we obtain that $\tilde{\theta} \approx 70^{\circ}$ K in this case.

Neglecting the under-recuperation at the warm end of the Joule – Thomson heat exchanger and the heat influx from the environment to the lower part of the refrigerator, the upper bound of the quantities M_3/M_2 can be obtained from (13) and (14) for $(Q_{bgd}+Q_{add})=0$.

The change in the ratio between the quantity of gas going into cooling the flow inlets and the quantity of gas supplied to the reverse refrigerator flow is shown in Fig. 3 as a function of the preliminary cooling in the refrigerator for several forward flow pressures and a 4.4°K cryostat temperature level.

NOTATION

x, axial flow inlet coordinate; l, flow inlet length; S, area of the current-carrying section; T, flow inlet temperature; T_{hot}, temperature of the hot end of the flow inlet; T_{cld}, temperature of the coldend of the flow inlet; T_{en}, environment temperature; T₁, preliminary cooling temperature; θ , temperature of the cooling gas; Q_r, refrigerator cold-productivity; ΣQ , total heat flux; Q_{bgd}, heat influx to the cryostat; q_{cld}, thermal flux density at the cold end of the flow inlet; Q_{add}, reserve of refrigerator cold-productivity; Q₃, cold used to cool the flow inlet; q_{en}, specific heat flux from the environment to the lower part of the refrigerator; l_r , specific power expenditure in the refrigerator; l_f , specific power expenditure in the refrigerator to obtain the fluid used after evaporation to cool the flow inlets; M₁, quantity or consumption of forward gas flow through the throttle valve; M₂, quantity or consumption of gas in reverse refrigerator flow; M_l, quantity of fluid or rate of liquefaction of the gas; M₃^{*}, quantity or consumption of gas at which the required refrigerator cold-productivity is minimal; r, heat of evaporation of liquid helium; c_p, specific heat of the cooling gas; I, electrical current intensity; j, current density; λ , coefficient of heat conduction; ρ , specific electrical resistivity; K^{*}_θ, axial temperature coefficient; i, specific enthalpy; s, specific entropy; ΔiT , isothermal effect of throttling; Δt , under recuperation; L, Lorentz constant; ε_C , cold coefficient of the Carnot cycle.

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A MODEL FOR CALCULATING THE ATTENUATION FACTOR OF A MULTICOMPONENT MEDIUM

E. Ya. Litovskii

A model and procedure are presented for calculating the attenuation factor of a multicomponent medium, taking account of the size distribution of particles and pores.

The attenuation factor is one of a number of important parameters of materials which are semitransparent to thermal radiation needed to estimate the radiation component of heat transfer, and to solve the radiation conduction heat-transfer equation. When the attenuation factor is known, radiation heat transfer in an optically thick layer of a heterogeneous medium can be estimated from the Rosseland formula [1]. Expressions are given in [1-4] and in other places for the attenuation factor of a two-component medium with identically sized particles.

In the present paper we propose a model for calculating the attenuation factor of two-component and multicomponent media which takes account of the size distribution of particles and pores.

To explain the proposed procedure, we consider in more detail the simplest model of a two-phase medium consisting of a regular chain of particles (plates) of the first phase of identical size d with the particles of the second phase in between. We choose a segment of arbitrary length L in the medium and consider the attenuation of radiation with an initial intensity I_0 in penetrating this segment. The attenuation occurs both because of true absorption and because of reflection from the phase boundaries. We assume that the reflection is specular and obeys the Fresnel formulas.

Suppose there are N particles of the first phase in the segment L. Then the number of reflections from them is 2N, and the total length of particles of the first phase is h = Nd. In this case

$$I = I_0 (1 - R)^{2N} \exp(-\alpha_1 h) \exp[-\alpha_2 (L - h)],$$
(1)

where α_1 and α_2 are the absorption coefficients of the first and second phases, respectively.

Introducing the effective attenuation factor Ke

$$I = I_0 \exp\left(-K_{\rm p}L\right),\tag{2}$$

equating the right-hand sides of Eqs. (1) and (2), and using the fact that $R \ll 1$, we obtain

$$K_{e} = \alpha_{2} + (\alpha_{1} - \alpha_{2})C + 2\frac{C}{d}R, \qquad (3)$$

where C = h/L = Nd/L is the concentration of particles of the first phase.

If $\alpha_2 = 0$, i.e., the second phase consists of pores, C = 1 - p, where p is the porosity,

$$K_{e} = (1 - p)(\alpha_{i} + 2R/d).$$
(4)

This relation is given in [5].

<u>Three-Phase System.</u> The particle sizes are identical and the third phase occupies the spaces between phases 1 and 2, e.g., a glassy phase in which there are crystalline precipitates,

$$K_{e} = \frac{2R_{1}}{d_{1}}C_{1} + \frac{2R_{2}}{d_{2}}C_{2} + \alpha_{3} + (\alpha_{1} - \alpha_{3})C_{1} + (\alpha_{2} - \alpha_{3})C_{2}.$$
 (5)

<u>n-Phase System</u>. The particle sizes in each phase are identical, and the n-th phase occupies the spaces between the remaining phases,

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